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Improved event-triggered control for a class of continuous-time switched linear systems

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Abstract: Both state-based and observer-based output feedback event-triggered controls for continuous-time switched linear systems are studied. An improved event-triggered sampling mechanism is adopted in feedback control, under which Zeno behaviour can be easily excluded in sampling process. Meanwhile, it is shown that the closed-loop switched system is stable in the sense of global uniform boundedness satisfying an average dwell time condition. A numerical example is finally given to verify the developed results.

1 Introduction

A switched system is usually composed of a family of continuous-time or discrete-time subsystems and a switching law which orchestrates switchings between the subsystems. The past two decades have seen lots of research activities focusing on the study of switched systems (see [1–11] and references therein) since many practical systems have to be modelled as switched systems, e.g. chemical processes, switched resistor–inductor–capacitor circuits and intelligent transportation systems. Most of the existing references study conventionally continuous feedback control of switched systems which requires signals to transmit continuously all the time. However, from a practical implementation point of view, controller is often implemented on a digital platform. Discrete signal transmission demonstrates its superiority due to the lower costs and the higher transfer efficiency. Periodic sampling and event-triggered sampling schemes are therefore used to control physical plants (see e.g. [12–15]). Event-triggered control is a control strategy in which control task is executed after the occurrence of an external event. Several event-triggered control strategies have been developed for different kinds of *non-switched* control systems (see e.g. [16–26]).

Recently, periodic sampling and quantised measurements method is developed to stabilise a switched linear control system in [27]. Wakaiki and Yamamoto [28] proposes a stability analysis method for periodically sampled-data switched linear systems with finite-level static quantisers. Fu *et al.* [29] extends the result of periodic sampling control to a switched linear neutral system. Periodic sampling control is in fact time-triggered control scheme, in which the inter-event times are constant. Fixed sampling period can help simplify the analysis of closed-loop system and the design of controller but may lead to a waste of computation resources regardless of the change in system operating. Event-triggered control can mitigate the unnecessary waste of computation resources and communication. The authors in [30, 31] propose event-triggered control of switched linear systems. Qi and Cao [32] considers finite-time event-triggered H_∞ control for switched systems with time-varying delay. Ma *et al.* [33] proposes event-triggered dynamic output feedback control schemes for switched linear systems. Although event-triggered control schemes of switched linear systems are studied in [30–33], positive lower bounds on inter-event interval derived from them are rough. In event-triggered control, the execution of control tasks occurs in non-fixed period and inter-event intervals are varying. Therefore, an infinite number of events may generate in finite time (the so called Zeno behaviour) [34]. Zeno behaviour needs to be avoided since it can make event-triggered control scheme infeasible for practice implementation [14].

Motivated by the above analysis, we study event-triggered control of continuous-time switched linear systems. The contribution of this paper lies in twofold. (i) We adapt an improved event-triggered control mechanism and apply it to a switched linear control system and derive a lower bound of minimum inter-event interval to exclude Zeno behaviour in event-triggered sampling process. (ii) We develop both state-based and observer-based output feedback controls for the switched control system to achieve its global uniform boundedness condition. The material in this paper was partially presented in a conference paper [35]. This paper corrects mistakes and completes results on stability analysis in [35] and further improves the structure of the paper.

The rest of the paper is organised as follows. Section 2 gives problem statement. Sections 3 and 4 construct event-triggered sampling mechanisms based on both system state and observer state and give stability analysis of closed-loop switched systems, respectively. A lower bound of the minimum inter-event interval is derived to guarantee that there is no Zeno behaviour occurring on event-triggered control process in Section 5. Section 6 presents a numerical example to illustrate the advantage of the proposed method. Finally, Section 7 concludes the whole paper.

Notations: Throughout the paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{N}$ denotes the set of non-negative integer numbers. For a square matrix P, P^{T} and P^{-1} denote the transpose and the inverse of P, respectively; $P > 0(< 0)$ means that the matrix P is real symmetric and positive definite (real symmetric and negative definite); $\underline{\lambda}(P)$ and $\bar{\lambda}(P)$ denote the minimum and the maximum eigenvalues of matrix P, respectively. $\| \cdot \|$ denotes the Euclidean vector norm. $\| P \|$ is spectral norm of matrix P. $\mathscr{B}(\varepsilon)$ is the closed ball in $\mathbb{R}^n$ with centre at the origin and radius ε. $*$ denotes the symmetry part of a symmetry matrix. $\mathrm{diag}\{\cdots\}$ denotes a block-diagonal matrix. The notations 0 and I denote a zero matrix and an identity matrix of appropriate dimensions, respectively.

2 Statement of the problem

Event-triggered control system considered in this paper is shown in Fig. 1.

The physical plant is continuous-time switched linear system

$$\begin{cases} \dot{x}(t) = A_\sigma x(t) + B_\sigma u_\sigma(t), \ x(0) = x_0 \\ y(t) = C_\sigma x(t) \end{cases} \tag{1}$$

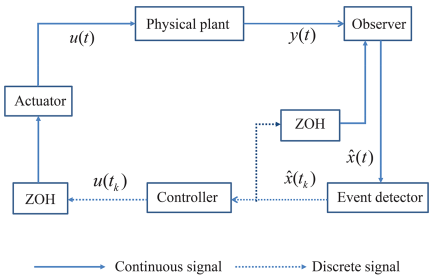

Fig. 1 *Event-triggered control system*

where $x(t) \in \mathbb{R}^n$ is system state, $y(t) \in \mathbb{R}^q$ is measurable output, $\sigma: [0, \infty) \to \mathscr{M} = \{1, 2, \ldots, m\}$ is a switching signal that orchestrates switching between subsystems, $\mathscr{M}$ is a finite index set, $A_i, B_i, C_i: i \in \mathscr{M}$ are known real matrices defining individual subsystem of system (1). $u_i \in \mathbb{R}^m$ is control input of subsystem i. It is assumed that pairs (A_i, B_i) and (A_i, C_i) are controllable and observable, respectively. Corresponding to the switching signal σ, there exists a switching sequence

$$\{x_{t_0}: (l_0, t_0), (l_1, t_1), \ldots, (l_i, t_i), \ldots \mid l_i \in \mathscr{M}, i \in \mathbb{N}\} \tag{2}$$

which means that the l_ith subsystem is active when $t \in [t_i, t_{i+1})$ and t_i is the switching instant. Without loss of generality, we assume that there are no state jumps at switching instants and the solution $x(\cdot)$ of system (1) is continuous everywhere.

Definition 1: Let $N_\sigma(s, t)$ denote the number of discontinuities of a switching signal σ on an interval (s,t) for $\forall t \geq s \geq 0$ [2].

(1) If any two switches are separated by at least $\tau_d > 0$, i.e. $N_\sigma(t, s) \leq 1$ when $t - s \leq \tau_d$, then τ_d is called a dwell time;
(2) If $N_\sigma(t, s) \leq N_0 + ((t - s)/\tau_a)$ holds for $\tau_a > \tau_d$ and $N_0 \geq 1$, then τ_a is called an average dwell time.

Denifition 2: Switched system (1) is uniformly bounded if for a constant $\delta > 0$, there exists a switching signal σ and a positive constant $\beta = \beta(\delta) < \infty$, which is independent of t_0, such that

$$\| x(t_0) \| < \delta \Rightarrow \| x(t) \| < \beta(\delta), \quad \forall t > t_0 . \tag{3}$$

Switched system (1) is globally uniformly bounded if (3) holds for $\forall \delta > 0$.

Lemma 1: For any real vectors u, ν and symmetric positive matrix Q with compatible dimension, the following inequality holds:

$$u^{\mathrm{T}}\nu + \nu^{\mathrm{T}}u \leq u^{\mathrm{T}}Qu + \nu^{\mathrm{T}}Q^{-1}\nu . \tag{4}$$

3 State-based event-triggered control

3.1 Sampling mechanism

We denote the instants when an event occurs by $\{\hat{t}_k\}_{k=0}^{\infty}$ with $\hat{t}_k < \hat{t}_{k+1}$ and $e(t) = x(t) - x(\hat{t}_k)$. The event-triggered condition is expressed in terms of the inequality

$$\| e(t) \|^2 \geq \eta \| x(t) \|^2 + \varepsilon \tag{5}$$

where ε and η are positive parameters. As shown in Fig. 1, event detector tests the triggering condition (5) continuously by receiving state from observer to determine whether an event is generated or not. Once an event is triggered, sampling occurs immediately. The sampling mechanism obtains the latest state information at sampling instants and then transmits it to controller. When an event happens, the error $e(t)$ is reset to zero and starts growing until it triggers a new measurement update. We assume that the first event is generated at time $\hat{t}_0 = t_0$. With the state $x(\hat{t}_k)$ sampled at time $\hat{t}_k$, the next sampling instant $\hat{t}_{k+1}$ is determined by

$$\hat{t}_{k+1} = \inf \{ t > \hat{t}_k \mid \| e(t) \|^2 = \eta \| x(t) \|^2 + \varepsilon \} . \tag{6}$$

Under the triggering condition (5), we construct a state feedback controller. Suppose that n samplings occur on interval $[t_i, t_{i+1})$ and $\hat{t}_{k+1}$ is the first sampling instant on this interval. According to switching sequence (2), without loss of generality, we assume that subsystem i is active on interval $[t_i, t_{i+1}]$, the piecewise continuous controller can be set as

$$u = u_i = \begin{cases} K_i x(\hat{t}_k), & t \in [t_i, \hat{t}_{k+1}) \\ K_i x(\hat{t}_{k+1}), & t \in [\hat{t}_{k+1}, \hat{t}_{k+2}) \\ \cdots & \\ K_i x(\hat{t}_{k+n}), & t \in [\hat{t}_{k+n}, t_{i+1}) \end{cases} \tag{7}$$

where K_i is the controller gain of subsystem i.

Remark 1: We assume that the controller possess same switching rule with switched system (1). Switching signal adopted here is time dependent and is known a priori. It is rational to construct controller (7) for subsystem i to form closed-loop system on $[t_i, t_{i+1}]$.

Remark 2: The plant and the controller switch synchronously since the switching rule depends on the dwell time. As shown in Fig. 1, event detector possesses three functions: detector, sampler and transmitter. The information of system state is sampled when an event is triggered and then transmitted to controller. Detector tests event-triggered condition continuously, but sampler and transmitter do not work if no event occurs. The feedback loop is in fact running in open-loop form during arbitrary two consecutive samplings.

Controller receives the sampled state $x(\hat{t}_k)$ at sampling instant $\hat{t}_k$ and holds it until the next event generates. On sampling interval $[\hat{t}_k, \hat{t}_{k+1})$, controller only computes at sampling instant $\hat{t}_k$. We therefore introduce a zero-order holder to keep the control signal continuous and assume that controller and actuator are collocated together.

Remark 3: Piecewise continuous controllers with various forms have been widely applied in continuous-time linear systems (see e.g. [36, 37]. While in this paper piecewise continuous controller is formed as a continuous step signal induced by event-triggered sampler and zero-order holder.

3.2 Stability analysis

According to switching sequence (2), we assume that subsystem i is active on $[t_i, t_{i+1})$. Suppose that n samplings occur on interval $[t_i, t_{i+1})$ and $\hat{t}_{k+1}$ is the first sampling instant on this interval. For $\forall t \in [t_i, \hat{t}_{k+1}), [\hat{t}_{k+1}, \hat{t}_{k+2}), \ldots, [\hat{t}_{k+n}, t_{i+1})$, error $e(t) = x(t) - x(\hat{t}_{k+j})$ holds for all $j = 0, 1, \ldots, n$. Substituting controller (7) into subsystem i yields

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i K_i x(\hat{t}_{k+j}) \\ &= A_i x(t) + B_i K_i (x(t) - e(t)) \\ &= (A_i + B_i K_i) x(t) - B_i K_i e(t) . \end{aligned} \tag{8}$$

The following theorem presents main result of this section.

Theorem 1: Consider system (1) with controller (7) determined by triggering condition (5). For given scalars $\eta > 0, \mu > 1, \varepsilon > 0$ and $N_0 \geq 1$, if there exist matrices $P_i > 0, P_j > 0$ and K_i for $\forall i, j \in \mathscr{M}$ such that

$$\begin{bmatrix} A_i^{\mathrm{T}}P_i + P_iB_iK_i + P_iA_i + K_i^{\mathrm{T}}B_i^{\mathrm{T}}P_i & P_iB_iK_i \\ * & -I \end{bmatrix} < 0 \tag{9}$$

and

$$P_i \le \mu P_j, \quad P_j \le \mu P_i \tag{10}$$

then system (1) is globally uniformly bounded for any switching signal with average dwell time satisfying $\tau_a > (\ln \mu)/\delta$, where $\delta > 0$ can be computed from the feasible matrix solutions of (9) and (10), and the state of system (1) exponentially converges to the bounded region

$$\mathscr{B}(\varepsilon) := \left\{ x(t) | \parallel x(t) \parallel \le \sqrt{\frac{\varepsilon e^{\delta\tau_a}\left((\mu-1)e^{\delta\tau_a N_0}+1\right)-\varepsilon\mu}{\min_{\forall i \in \mathscr{M}}(\underline{\lambda}(P_i))\delta(e^{\delta\tau_a}-\mu)}} \right\}. \tag{11}$$

Proof: Construct piecewise Lyapunov function candidate

$$V_i(t) = x^{\mathrm{T}}(t)P_ix(t) \tag{12}$$

where $P_i > 0$. For $t \in [t_i, t_{i+1})$, differentiating (12) along solutions of system (8) and using Lemma 1 and condition (5) gives

$$\begin{aligned} \dot{V}_i(t) &= 2[(A_i + B_iK_i)x(t) - B_iK_ie(t)]^{\mathrm{T}}P_ix(t) \\ &= 2x^{\mathrm{T}}(t)(A_i + B_iK_i)^{\mathrm{T}}P_ix(t) - 2e^{\mathrm{T}}(t)K_i^{\mathrm{T}}B_i^{\mathrm{T}}P_ix(t). \\ &\le 2x^{\mathrm{T}}(t)(A_i + B_iK_i)^{\mathrm{T}}P_ix(t) + e^{\mathrm{T}}(t)e(t) \\ &\quad + x^{\mathrm{T}}(t)P_iB_iK_iK_i^{\mathrm{T}}B_i^{\mathrm{T}}P_ix(t) \\ &\le x^{\mathrm{T}}(t)Q_ix(t) + \varepsilon \end{aligned} \tag{13}$$

where $Q_i = A_i^{\mathrm{T}}P_i + K_i^{\mathrm{T}}B_i^{\mathrm{T}}P_i + P_iA_i + P_iB_iK_iK_i^{\mathrm{T}}B_i^{\mathrm{T}}P_i + \eta I \quad + P_iB_iK_i$. By utilising Schur complement lemma, inequality (9) is equivalent to $Q_i < 0$. Thus, we have from (13) that

$$\begin{aligned} \dot{V}_i(t) &\le -\lambda_{\min}(-Q_i) \parallel x(t) \parallel^2 + \varepsilon \\ &\le -\frac{\lambda_{\min}(-Q_i)}{\lambda_{\max}(P_i)}x^{\mathrm{T}}(t)P_ix(t) + \varepsilon \\ &= -\delta_iV_i(t) + \varepsilon \end{aligned} \tag{14}$$

where $\delta_i = (\lambda_{\min}(-Q_i))/(\lambda_{\max}(P_i)) > 0$. Integrating (14) from t_i to t yields

$$V_i(t) \le e^{-\delta_i(t-t_i)}V_i(t_i) + \varepsilon\int_{t_i}^{t} e^{-\delta_i(t-s)}\,\mathrm{d}s. \tag{15}$$

Moreover, inequality (10) implies that $V_{l_i}(t_i) \le \mu V_{l_{i-1}}(t_i^-)$ for $\forall l_i, l_{i-1} \in \mathscr{M}$. Let $l_{t_i} = l(t_i, t) = N_\sigma(t_i, t) \le ((t - t_i)/\tau_a) + N_0$ and $\delta = \min_{\forall l_i \in \mathscr{M}} \delta_{l_i} > 0$. From (15), we have

$$\begin{aligned} V_\sigma(t) = \; & V_{l_i}(t) \le e^{-\delta(t-t_i)}\mu V_{l_{i-1}}(t_i^-) + \frac{\varepsilon}{\delta}\left(1 - e^{-\delta(t-t_i)}\right) \\ &\le e^{-\delta(t-t_i)}\mu\Big(e^{-\delta(t_i-t_{i-1})}V_{l_{i-1}}(t_{i-1}) \\ &\quad + \frac{\varepsilon}{\delta}\left(1 - e^{-\delta(t_i-t_{i-1})}\right)\Big) + \frac{\varepsilon}{\delta}\left(1 - e^{-\delta(t-t_i)}\right) \\ &\le e^{-\delta(t-t_{i-1})}\mu V_{l_{i-1}}(t_{i-1}) \\ &\quad + \frac{\varepsilon\mu}{\delta}\left(e^{-\delta(t-t_i)} - e^{-\delta(t-t_{i-1})}\right) + \frac{\varepsilon}{\delta}\left(1 - e^{-\delta(t-t_i)}\right) \\ &\le e^{-\delta(t-t_{i-1})}\mu^2 V_{l_{i-2}}(t_{i-1}^-) \\ &\quad + \frac{\varepsilon\mu}{\delta}\left(e^{-\delta(t-t_i)} - e^{-\delta(t-t_{i-1})}\right) + \frac{\varepsilon}{\delta}\left(1 - e^{-\delta(t-t_i)}\right) \\ &\le \cdots \le e^{-\delta(t-t_0)}\mu^{l_{t_0}}V_{l_0}(t_0) + \frac{\varepsilon\mu^{l_{t_1}}}{\delta}\left(e^{-\delta(t-t_2)} - e^{-\delta(t-t_1)}\right) \\ &\quad + \frac{\varepsilon\mu^{l_{t_2}}}{\delta}\left(e^{-\delta(t-t_3)} - e^{-\delta(t-t_2)}\right) \\ &\quad + \cdots + \frac{\varepsilon\mu^2}{\delta}\left(e^{-\delta(t-t_{i-1})} - e^{-\delta(t-t_{i-2})}\right) \\ &\quad + \frac{\varepsilon\mu}{\delta}\left(e^{-\delta(t-t_i)} - e^{-\delta(t-t_{i-1})}\right) + \frac{\varepsilon}{\delta}\left(1 - e^{-\delta(t-t_i)}\right) \\ &\le e^{-\delta(t-t_0)}\mu^{l_{t_0}}\left(V_{l_0}(t_0) - \frac{\varepsilon}{\delta\mu}\right) \\ &\quad + \frac{\varepsilon(\mu-1)}{\delta}\sum_{m=0}^{l_{t_2}}\mu^m e^{-\delta(t-t_{i-m})} + \frac{\varepsilon}{\delta} \\ &\le \mu^{N_0}e^{-(\delta-(\ln\mu/\tau_a))(t-t_0)}\left(V_{l_0}(t_0) - \frac{\varepsilon}{\delta\mu}\right) \\ &\quad + \frac{\varepsilon(\mu-1)}{\delta}e^{\delta\tau_a N_0}\sum_{m=0}^{l_{t_2}}e^{m(\ln\mu-\delta\tau_a)} + \frac{\varepsilon}{\delta} \end{aligned} \tag{16}$$

where τ_d is the minimum dwell time. From Lyapunov function (12), we have

$$\begin{aligned} V_{l_i}(t) = \; & x^{\mathrm{T}}(t)P_{l_i}x(t) \ge \underline{\lambda}(P_{l_i}) \parallel x(t) \parallel^2 \\ & \ge \min_{\forall l_i \in \mathscr{M}}(\underline{\lambda}(P_{l_i})) \parallel x(t) \parallel^2 = \alpha \parallel x(t) \parallel^2 \end{aligned} \tag{17}$$

and

$$V_{\sigma(t_0)}(t_0) \le \max_{\forall l_i \in \mathscr{M}}(\bar{\lambda}(P_{l_i})) \parallel x(t_0) \parallel^2 = \beta \parallel x(t_0) \parallel^2 \tag{18}$$

where $\alpha = \min_{\forall l_i \in \mathscr{M}}(\underline{\lambda}(P_{l_i})), \beta = \max_{\forall l_i \in \mathscr{M}}(\bar{\lambda}(P_{l_i}))$. Thus, combining (16) with (17) and (18), we have

$$\begin{aligned} \parallel x(t) \parallel^2 &\le \frac{1}{\alpha}V_{l_i}(t) \\ &\le \frac{\beta}{\alpha}\mu^{N_0}e^{-(\delta-((\ln\mu)/\tau_a))(t-t_0)}\left(\parallel x(t_0) \parallel^2 - \frac{\varepsilon}{\delta\mu\beta}\right) \\ &\quad + \frac{\varepsilon(\mu-1)}{\alpha\delta}e^{\delta\tau_a N_0}\sum_{m=0}^{l_{t_2}}e^{m(\ln\mu-\delta\tau_a)} + \frac{\varepsilon}{\alpha\delta}. \end{aligned} \tag{19}$$

The condition $\tau_a > (\ln \mu)/\delta$ in Theorem 1 implies that $\delta - ((\ln \mu)/\tau_a) > 0$ and $\ln \mu - \delta\tau_a < 0$. Then from (19), we have

$$\begin{aligned} \parallel x(t) \parallel^2 &\le \frac{\beta}{\alpha}\mu^{N_0}e^{-(\delta-((\ln\mu)/\tau_a))(t-t_0)}\left(\parallel x(t_0) \parallel^2 - \frac{\varepsilon}{\delta\mu\beta}\right) \\ &\quad + \frac{\varepsilon(\mu-1)e^{\delta\tau_a(N_0+1)}}{\alpha\delta(e^{\delta\tau_a}-\mu)} + \frac{\varepsilon}{\alpha\delta}, \end{aligned} \tag{20}$$

under which uniform boundedness of system (8) can be guaranteed.□

Remark 4: If $\varepsilon = 0$ in (5), then the result of Theorem 1 can be degenerated to the following corollary.

Corollary 1: Consider system (1) with controller (7) determined by triggering condition (5) with $\varepsilon = 0$. For given scalars $\eta > 0$ and $\mu > 1$, if there exist matrices $P_i > 0, P_j > 0$ and K_i for $\forall i, j \in \mathscr{M}$ such that inequalities (9) and (10) hold, then system (1) is exponentially stable for any switching signal with average dwell time satisfying $\tau_a > (\ln \mu)/\delta$, where $\delta > 0$ can be computed from the feasible matrix solutions of (9) and (10).

4 Observer-based event-triggered control

4.1 Asymptotic observer

For subsystem i, we construct observer

$$\dot{\hat{x}}(t) = A_i\hat{x}(t) + B_iu_i(t) + L_i(y(t) - C_i\hat{x}(t)) \tag{21}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is observer state, L_i is observer gain of subsystem i. Suppose that state observer is collocated with sensor and multiple sub-observers possess the same switching law with system (1). We form switching observer

$$\dot{\hat{x}}(t) = A_\sigma\hat{x}(t) + B_\sigma u_\sigma(t) + L_\sigma C_\sigma(x(t) - \hat{x}(t)). \tag{22}$$

Let error be $e(t) = x(t) - \hat{x}(t)$. Then error system

$$\dot{e}(t) = (A_\sigma - L_\sigma C_\sigma)e(t) \tag{23}$$

can be established from (1) and (22).

The following lemma presents design of asymptotic observer.

Lemma 2: Consider system (23). For $\forall i \in \mathscr{M}$, if there exist matrices $R_i > 0$ and X_i satisfying

$$A_i^{\mathrm{T}}R_i - C_i^{\mathrm{T}}X_i^{\mathrm{T}} + R_iA_i - X_iC_i < 0, \tag{24}$$

then (21) is an asymptotic observer of subsystem i and gain L_i is given by $L_i = R_i^{-1}X_i$.

4.2 Sampling mechanism

We assume that event detector and sampler are collocated in a same node and denote the instants when an event happens by $\{\hat{t}_k\}_{k=0}^{\infty}$ with $\hat{t}_k < \hat{t}_{k+1}$. Let $\hat{e}(t) = \hat{x}(t) - \hat{x}(\hat{t}_k)$. Construct event-triggered condition

$$\| \hat{e}(t) \|^2 \ge \eta \| \hat{x}(t) \|^2 + \varepsilon \tag{25}$$

where ε and η are positive parameters. With the state $\hat{x}(\hat{t}_k)$ sampled at time $\hat{t}_k$, the next sampling instant $\hat{t}_{k+1}$ can be determined by

$$\hat{t}_{k+1} = \inf \{t > \hat{t}_k | \| \hat{e}(t) \|^2 = \eta \| \hat{x}(t) \|^2 + \varepsilon\}. \tag{26}$$

Under the triggering condition (25), we construct an observer-based feedback controller. Suppose that n samplings occur on interval $[t_i, t_{i+1})$ and $\hat{t}_{k+1}$ is the first sampling instant on this interval. According to switching sequence (2), we set piecewise continuous controller

$$u = u_i = \begin{cases} K_i\hat{x}(\hat{t}_k),\ t \in [t_i, \hat{t}_{k+1}) \\ K_i\hat{x}(\hat{t}_{k+1}),\ t \in [\hat{t}_{k+1}, \hat{t}_{k+2}) \\ \cdots \\ K_i\hat{x}(\hat{t}_{k+n}),\ t \in [\hat{t}_{k+n}, t_{i+1}) \end{cases} \tag{27}$$

where K_i is the controller gain of subsystem i.

4.3 Stability analysis

According to switching sequence (2), subsystem l_i is active on $[t_i, t_{i+1})$. Suppose that n samplings occur on interval $[t_i, t_{i+1})$ and $\hat{t}_{k+1}$ is the first sampling instant on this interval. For $\forall t \in [t_i, \hat{t}_{k+1}), [\hat{t}_{k+1}, \hat{t}_{k+2}), \ldots, [\hat{t}_{k+n}, t_{i+1})$, error $\hat{e}(t) = \hat{x}(t) - \hat{x}(\hat{t}_{k+j})$ holds for all $j = 0, \ldots, n$. Substituting controller (27) into (21) yields

$$\begin{aligned} \dot{\hat{x}}(t) &= A_i\hat{x}(t) + B_iK_i\hat{x}(\hat{t}_{k+j}) + L_iC_ie(t) \\ &= A_i\hat{x}(t) + B_iK_i(\hat{x}(t) - \hat{e}(t)) + L_iC_ie(t) \\ &= (A_i + B_iK_i)\hat{x}(t) + L_iC_ie(t) - B_iK_i\hat{e}(t). \end{aligned} \tag{28}$$

Together with (23) and (28), for $t \in [t_i, t_{i+1})$, we have

$$\begin{cases} \dot{\hat{x}}(t) = (A_i + B_iK_i)\hat{x}(t) + L_iC_ie(t) - B_iK_i\hat{e}(t) \\ \dot{e}(t) = (A_i - L_iC_i)e(t). \end{cases} \tag{29}$$

We know from $e(t) = x(t) - \hat{x}(t)$ that system (1) is stable under feedback control (29) if and only if augmented system

$$\begin{cases} \dot{\hat{x}}(t) = (A_\sigma + B_\sigma K_\sigma)\hat{x}(t) + L_\sigma C_\sigma e(t) - B_\sigma K_\sigma\hat{e}(t) \\ \dot{e}(t) = (A_\sigma - L_\sigma C_\sigma)e(t) \end{cases} \tag{30}$$

is stable. The compact form of (30) can be written as

$$\dot{\xi}(t) = \bar{A}_\sigma\xi(t) + \bar{B}_\sigma\tilde{e}(t) \tag{31}$$

where

$$\xi(t) = \begin{bmatrix} \hat{x}(t) \\ e(t) \end{bmatrix},\quad \bar{A}_\sigma = \begin{bmatrix} A_\sigma + B_\sigma K_\sigma & L_\sigma C_\sigma \\ 0 & A_\sigma - L_\sigma C_\sigma \end{bmatrix},$$
$$\bar{B}_\sigma = \begin{bmatrix} -B_\sigma K_\sigma & 0 \\ 0 & 0 \end{bmatrix},\quad \tilde{e}(t) = \begin{bmatrix} \hat{e}(t) \\ 0 \end{bmatrix}.$$

We thus focus on system (31). The following theorem gives main result of this section.

Theorem 2: Consider system (31) with sampling instants determined by (25). For given scalars $\eta > 0, \mu > 1, \varepsilon > 0$ and $N_0 \ge 1$, if there exist matrices $P_i > 0, P_j > 0, K_i$ and L_i for $\forall i, j \in \mathscr{M}$ satisfying

$$\begin{bmatrix} \hat{Q}_i^{11} & P_iL_iC_i & P_iB_iK_i \\ * & \hat{Q}_i^{22} & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{32}$$

and

$$P_i \le \mu P_j,\quad P_j \le \mu P_i \tag{33}$$

where

$$\begin{aligned} \hat{Q}_i^{11} &= A_i^{\mathrm{T}}P_i + K_i^{\mathrm{T}}B_i^{\mathrm{T}}P_i + P_iB_iK_i + P_iA_i, \\ \hat{Q}_i^{22} &= P_iA_i - P_iL_iC_i + A_i^{\mathrm{T}}P_i - C_i^{\mathrm{T}}L_i^{\mathrm{T}}P_i, \end{aligned}$$

then system (31) is globally uniformly bounded for any switching signal with average dwell time satisfying $\tau_{\mathrm{a}} > (\ln \mu)/\delta$, where $\delta > 0$ can be computed from feasible matrix solutions of (32) and (33), and the state of system (31) exponentially converges to the bounded region

$$\mathscr{B}(\varepsilon) := \left\{ \xi(t) | \ \| \xi(t) \| \ \le \sqrt{\frac{\varepsilon e^{\delta\tau_{\mathrm{a}}}\left((\mu - 1)e^{\delta\tau_{\mathrm{a}}N_0} + 1\right) - \varepsilon\mu}{\min_{\forall i \in \mathscr{M}} (\underline{\lambda}(P_i))\delta(e^{\delta\tau_{\mathrm{a}}} - \mu)}} \right\}. \tag{34}$$

Proof: The proof is analogous to the one of Theorem 1, thus it is omitted. □

5 Minimum inter-event interval

Event-triggered control brings more complicated dynamic behaviour to switched systems than time-triggered control. In event-triggered control, the execution of control tasks occurs aperiodically and inter-event intervals are varying. Therefore, we

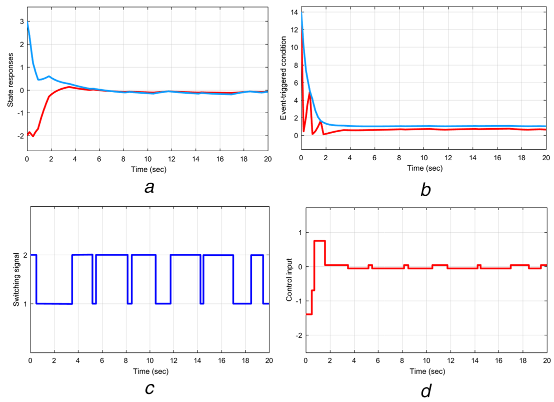

Fig. 2 *Simulation results with event-triggered condition* $\| e(t) \|^2 \geq \| x(t) \|^2 + 1$
(a) State responses, ***(b)*** Event-triggered condition, ***(c)*** Switching signal, ***(d)*** Control input

need to show that there always exists a non-zero lower bound of the minimum inter-event interval for event-triggered condition to exclude Zeno behaviour. In order to highlight the proof process, we only consider observer-based event-triggered control. The proof of the lower bound of minimum inter-event interval in state-based event-triggered control can be easily deduced from the following analysis.

Theorem 3: With event-triggered condition (25), the minimum inter-event interval is lower bounded by a positive scalar.

Proof: Suppose that n samplings happen on interval $[t_i, t_{i+1})$ and $\hat{t}_{k+1}, \ldots, \hat{t}_{k+n}$ are n sampling instants, respectively. On sampling intervals $[t_i, \hat{t}_{k+1})$, $[\hat{t}_{k+1}, \hat{t}_{k+2}), \ldots, [\hat{t}_{k+n}, t_{i+1})$, no matter which sampling interval t belongs to, $\hat{x}(\hat{t}_{k+l})$ are constants and $\hat{e}(t) = \hat{x}(t) - \hat{x}(\hat{t}_{k+l})$, where $l = 0, 1, \ldots, n$. Then, for $\forall t \in [t_i, \hat{t}_{k+1}), [\hat{t}_{k+1}, \hat{t}_{k+2}), \ldots, [\hat{t}_{k+n}, t_{i+1})$, we have

$$\begin{aligned}\dot{\hat{e}}(t) = &\ \dot{\hat{x}}(t) = A_i\hat{x}(t) + B_iK_i\hat{x}(\hat{t}_{k+l}) + L_iC_ie(t)\\ = &\ A_i\hat{e}(t) + (A_i + B_iK_i)\hat{x}(\hat{t}_{k+l}) + L_iC_ie(t)\,.\end{aligned} \tag{35}$$

Hence

$$\begin{aligned}\hat{e}(t) = &\ e^{A_i(t - \hat{t}_{k+l})}\hat{e}(\hat{t}_{k+l})\\ &+ \int_{\hat{t}_{k+l}}^{t} e^{A_i(t-s)}\Big((A_i + B_iK_i)\hat{x}(\hat{t}_{k+l}) + L_iC_ie(s)\Big)\,\mathrm{d}s\,.\end{aligned} \tag{36}$$

Since $\hat{e}(\hat{t}_{k+l}) = \hat{x}(\hat{t}_{k+l}) - \hat{x}(\hat{t}_{k+l}) = 0$, then

$$\hat{e}(t) = \int_{\hat{t}_{k+l}}^{t} e^{A_i(t-s)}\Big((A_i + B_iK_i)\hat{x}(\hat{t}_{k+l}) + L_iC_ie(s)\Big)\,\mathrm{d}s\,. \tag{37}$$

Therefore

$$\begin{aligned}\| \hat{e}(t) \| \leq &\int_{\hat{t}_{k+l}}^{t} e^{\| A_i \| (t-s)}\big(\| (A_i + B_iK_i) \| \ \| \hat{x}(\hat{t}_{k+l}) \| \\ &+ \| L_iC_i \| \ \| e(s) \| \big)\,\mathrm{d}s\\ \leq &\int_{\hat{t}_{k+l}}^{t} e^{\| A_i \| (t-s)}\big(\| (A_i + B_iK_i) \| \ \| \hat{x}(\hat{t}_{k+l}) \| \\ &+ \| L_iC_i \| \ \| e^{\sum_{m=1}^{i}(A_m - L_mC_m)s} \| \ \| e(0) \| \big)\,\mathrm{d}s\,.\end{aligned} \tag{38}$$

Noticing that $A_m - L_mC_m$ are Hurwitz for $\forall m = 1, \ldots, i$ with $\lambda_{\max}(A_m - L_mC_m) < 0$, we have

$$\begin{aligned}\| \hat{e}(t) \| \leq &\int_{\hat{t}_{k+l}}^{t} e^{\| A_i \| (t-s)}\big(\| (A_i + B_iK_i) \| \ \| \hat{x}(\hat{t}_{k+l}) \| \\ &+ e^{\sum_{m=1}^{i}((\lambda_{\max}(A_m - L_mC_m))/2)s} \| L_iC_i \| \ \| e(0) \| \big)\,\mathrm{d}s\\ \leq &\ \phi(\hat{t}_{k+l}) \int_{\hat{t}_{k+l}}^{t} e^{\| A_i \| (t-s)}\,\mathrm{d}s\end{aligned} \tag{39}$$

where

$$\begin{aligned}\phi(\hat{t}_{k+l}) = &\ \| (A_i + B_iK_i) \| \ \| \hat{x}(\hat{t}_{k+l}) \| \\ &+ e^{\sum_{m=1}^{i}((\lambda_{\max}(A_m - L_mC_m))/2)\hat{t}_{k+l}} \| L_iC_i \| \ \| e(0) \| \,.\end{aligned}$$

If $\| A_i \| \neq 0$, we have

$$\| \hat{e}(t) \| \leq \frac{\phi(\hat{t}_{k+l})}{\| A_i \|}\Big(e^{\| A_i \| (t - \hat{t}_{k+l})} - 1\Big). \tag{40}$$

According to (25), the next event will be generated when $\| \hat{e}(t) \|^2 = \eta \| \hat{x}(t) \|^2 + \varepsilon$. Hence, a lower bound on inter-event interval denoted by $T = t - \hat{t}_{k+l}$ can be determined by

$$\frac{\phi(\hat{t}_{k+l})}{\| A_i \|}\big(e^{\| A_i \| T} - 1\big) = \sqrt{\eta \| \hat{x}(t) \|^2 + \varepsilon}\,. \tag{41}$$

Thus

$$T = \frac{1}{\| A_i \|}\ln\left(\frac{\| A_i \| \sqrt{\eta \| \hat{x} \|^2 + \varepsilon}}{\phi(\hat{t}_{k+l})} + 1\right) \tag{42}$$

which means that for any given sampling instant $\hat{t}_{k+l}, T > 0$. If $\| A_i \| = 0$, we have

$$T\phi(\hat{t}_{k+l}) = \sqrt{\eta \| \hat{x}(t) \|^2 + \varepsilon} \tag{43}$$

which also indicates that $T > 0$. With the above discussion, it can be concluded that there always exists a positive lower bound of the minimum inter-event interval for event-triggered condition (25). □

6 Numerical example

In this section, we present a numerical example based on state-based event-triggered control method to show the advantage of the proposed method.

Example: Consider system (1) with two subsystems, where

$$A_1 = \begin{bmatrix} -0.4 & 0 \\ 0.7 & -0.7 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -0.6 & 0.5 \\ 0.3 & -0.6 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Let parameters be $\eta = 1$ and $\mu = 1.1$. Choose $K_1 = [-0.8 \quad -1]$ and $K_2 = [-0.7 \quad -0.7]$. By solving inequalities (8) and (9) in Theorem 1, we obtain

$$P_1 = \begin{bmatrix} 0.9546 & -0.2768 \\ -0.2768 & 0.8332 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.9442 & -0.2602 \\ -0.2602 & 0.8677 \end{bmatrix},$$
$$Q_1 = \begin{bmatrix} -0.4823 & 0.3833 \\ 0.3833 & -0.7715 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} -0.7883 & 0.5479 \\ 0.5479 & -0.7903 \end{bmatrix}.$$

δ is computed by

$$\delta = \min_{i=1,2}\left(\frac{\lambda_{\min}(-Q_i)}{\lambda_{\max}(P_i)}\right) = 0.1864$$

form the feasible solutions, and then the minimum average dwell time is obtained by $\tau_a^* = (\ln \mu)/\delta = 0.5113$. Choose a switching sequence $\tau_a > \tau_a^*$ and an initial state $x_0 = [-2\ 3]^T$. In order to show

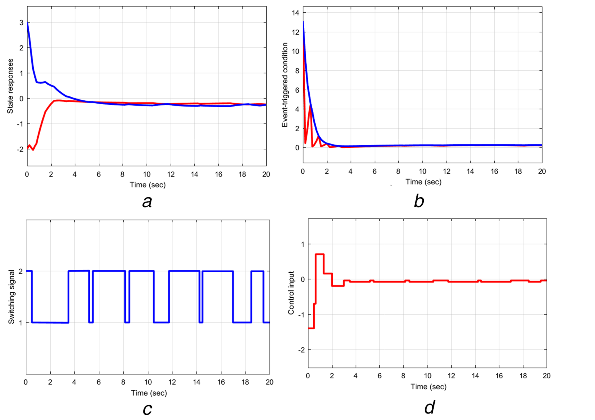

Fig. 3 *Simulation results with event-triggered condition* $\| e(t) \|^2 \geq \| x(t) \|^2 + 0.1$
(a) State responses, ***(b)*** Event-triggered condition, ***(c)*** Switching signal, ***(d)*** Control input

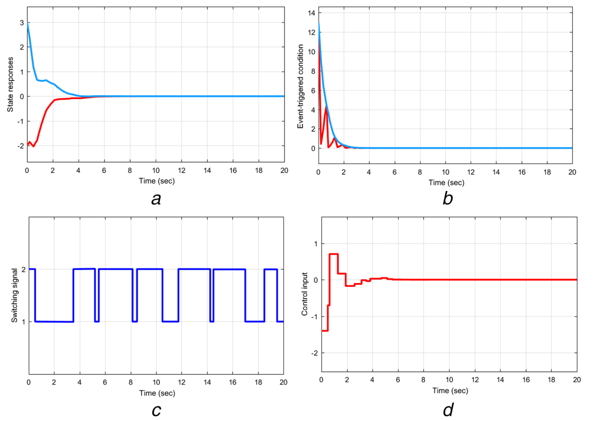

Fig. 4 *Simulation results with event-triggered condition* $\| e(t) \|^2 \geq \| x(t) \|^2$
(a) State responses, ***(b)*** Event-triggered condition, ***(c)*** Switching signal, ***(d)*** Control input

the advantage of the proposed method, we take $\varepsilon = 1, 0.1, 0$, successively.

Under event-triggered sampling mechanism (4) with different values of ε and a common switching rule satisfying $\tau_a > \tau_a^*$, we obtain simulation results shown in Figs. 2–4, respectively, from which, we can see that sampling times reduce when ε becomes large and state trajectories are influenced by the change of ε. Therefore, selecting appropriate parameter ε can save the cost while maintaining the performance of closed-loop system.

7 Conclusions

An improved event-triggered sampling mechanism is adopted into controller design for switched linear systems and a globally uniformly bounded condition of the closed-loop switched system is achieved in the framework of average dwell time technique. A lower bound of the minimum inter-event interval is derived to avoid Zeno behaviour in event-triggered sampling process.

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9 References

[1] Branicky, M.: 'Multiple Lyapunov functions and other analysis tools for switched and hybrid systems', *IEEE Trans. Autom. Control*, 1998, **43**, pp. 475–482
[2] Hespanha, J.P., Morse, A.S.: 'Stability of switched systems with average dwell-time'. Proc. 38th IEEE Conf. Decision Control, vol. 3, 1999, pp. 2655–2660
[3] Liberzon, D.: '*Switched in systems and control*' (Birkhauser, Boston, 2003)
[4] Sun, Z., Ge, S.S.: 'Analysis and synthesis of switched linear control systems', *Automatica*, 2005, **41**, pp. 181–195
[5] Sun, X., Zhao, J., Hill, D.: 'Stability and L_2-gain analysis for switched delay system: a delay-dependent method', *Automatica*, 2006, **42**, pp. 1769–1774
[6] Zhao, J., Hill, D.: 'On stability, L_2-gain and H_∞ control for switched systems', *Automatica*, 2008, **44**, pp. 1220–1232
[7] Lin, H., Antsaklis, P.J.: 'Stability and stabilizability of switched linear systems: a survey of recent results', *IEEE Trans. Autom. Control*, 2009, **54**, pp. 308–322
[8] Ma, R., Zhao, J.: 'Backstepping design for global stabilization of switched nonlinear systems in lower triangular form under arbitrary switchings', *Automatica*, 2010, **46**, pp. 1819–1823
[9] Fu, J., Ma, R., Chai, T.: 'Global finite-time stabilization of a class of switched nonlinear systems with the powers of positive odd rational numbers', *Automatica*, 2015, **54**, pp. 360–373
[10] Niu, B., Liu, Y., Zong, G., *et al.*: 'Command filter-based adaptive neural tracking controller design for uncertain switched nonlinear output-constrained systems', *IEEE Trans. Cybern.*, 2017, **99**, pp. 1–12
[11] Niu, B., Karimi, H.R., Wang, H., *et al.*: 'Adaptive output-feedback controller design for switched nonlinear stochastic systems with a modified average dwell-time method', *IEEE Trans. Syst. Man Cybern., Syst.*, 2017, **47**, pp. 1371–1382
[12] Chen, T., Francis, B.A.: '*Optimal sampled-data control systems*' (Springer Science & Business Media, London, 2012)
[13] Cassandras, C.: 'Event-driven control, communication, and optimization'. Proc. 32nd Chinese Control Conf., 2013, pp. 1–5
[14] Borgers, D.P., Heemels, W.P.M.H.: 'Event-separation properties of event-triggered control systems', *IEEE Trans. Autom. Control*, 2014, **59**, pp. 2644–2656
[15] Lunze, J., Lehmann, D.: 'A state-feedback approach to event-based control', *Automatica*, 2010, **46**, pp. 211–215
[16] Lehmann, D., Lunze, J.: 'Event-based output-feedback control'. Proc. 19th Mediterranean Conf. Control Automation, 2011, pp. 982–987
[17] Astrom, K.J., Bernhardsson, B.: 'Comparison of periodic and event based sampling for first-order stochastic systems'. Proc. 14th IFAC World Congress, 1999, pp. 301–306
[18] Wang, X., Lemmon, M.: 'Event design in event-triggered feedback control systems'. Proc. 47th IEEE Conf. Decision Control, 2008, pp. 2105–2110
[19] Wang, X., Lemmon, M.D.: 'Self-triggered feedback control systems with finite-gain stability', *IEEE Trans. Autom. Control*, 2009, **54**, pp. 452–467
[20] Heemels, W.P.M.H., Donkers, M.C.F.: 'Model-based periodic event-triggered control for linear systems', *Automatica*, 2013, **49**, pp. 698–711
[21] Zhang, J., Feng, G.: 'Event-driven observer-based output feedback control for linear systems', *Automatica*, 2014, **50**, pp. 1852–1859
[22] Donkers, M.C.F., Heemels, W.P.M.H.: 'Output-based event-triggered control with guaranteed l_∞-gain and improved and decentralized event-triggering', *IEEE Trans. Autom. Control*, 2012, **57**, pp. 1362–1376
[23] Persis, C.D., Tesi, P.: 'Resilient control under denial-of-service', *IFAC Proc. Vol.*, 2014, **47**, pp. 134–139
[24] Persis, C.D., Tesi, P.: 'Input-to-state stabilizing control under denial-of-service', *IEEE Trans. Autom. Control*, 2015, **60**, pp. 2930–2944
[25] Liu, J., Zha, L., Cao, J., *et al.*: 'Hybrid-driven-based stabilisation for networked control systems', *IET Control Theory Applic.*, 2016, **10**, pp. 2279–2285
[26] Liu, J., Xia, J., Tian, E., *et al.*: 'Hybrid-driven-based H_∞ filter design for neural networks subject to deception attacks', *Appl. Math. Comput.*, 2018, **320**, pp. 158–174
[27] Liberzon, D.: 'Finite data-rate feedback stabilization of switched and hybrid linear systems', *Automatica*, 2014, **50**, pp. 409–420
[28] Wakaiki, M., Yamamoto, Y.: 'Stability analysis of sampled-data switched systems with quantization', *Automatica*, 2016, **69**, pp. 157–168
[29] Fu, J., Li, T.F., Chai, T., *et al.*: 'Sampled-data-based stabilization of switched linear neutral systems', *Automatica*, 2016, **72**, pp. 92–99
[30] Wang, X., Ma, D.: 'Event-triggered control for continuous-time switched systems'. Proc. 27th Chinese Control Decision Conf., 2015, pp. 1143–1148
[31] Li, T.F., Fu, J.: 'Event-triggered control of switched linear systems', *J. Franklin Inst.*, 2017, **354**, pp. 6451–6462
[32] Qi, Y., Cao, M.: 'Event-triggered dynamic output feedback control for switched linear systems'. Proc. 35th Chinese Control Conf., 2016, pp. 2361–2367
[33] Ma, G., Liu, X., Qin, L., *et al.*: 'Finite-time event-triggered H_∞ control for switched systems with time-varying delay', *Neurocomputing*, 2016, **207**, pp. 828–842
[34] Lamperski, A., Ames, A.D.: 'Lyapunov theory for Zeno stability', *IEEE Trans. Autom. Control*, 2013, **58**, pp. 100–112
[35] Li, T.F., Fu, J.: 'Observer-based dynamic output event-triggered control of switched systems'. Proc. 36th Chinese Control Conf., 2017
[36] Xie, X., Lam, J., Li, P.: 'Finite-time H_∞ control of periodic piecewise linear systems', *Int. J. Syst. Sci.*, 2017, **48**, pp. 2333–2344
[37] Tan, F., Zhou, B., Duan, G.: 'Finite-time stabilization of linear time-varying systems by piecewise constant feedback', *Automatica*, 2016, **68**, pp. 277–285